Design and Analyze the Batch Ordering Supply Chain System

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Abstract—The design and management of a supply chain system have already attracted many attentions among process system engineering researchers recently. One of these areas is the analysis of logistic management for a supply chain using the system control theory. A supply chain can be viewed as a discrete system with lead times and operating constraints. In this paper, we use material and information balances to design a discrete dynamic model for a batch ordering supply chain system. The explicit transfer function model of the closed loop response is obtained by $z$-transform. The entire chain can be modeled by connecting these transfer functions in to a block diagram. In order to derive the $z$-transform model of the batch ordering system, we first use the signal processing technique. The model proves to be a very powerful tool that reveals the dynamics characteristic of the system. Moreover, the use of the dynamic process control theory allows us to design the inventory level control strategies and analyze its dynamics behaviors. Based on the transfer functions in $z$-transform, it becomes possible to explore the stability of a supply chain. Furthermore, we can show the effective reduction of the bullwhip effect and improvement of customer satisfaction for a batch ordering supply chain by implementing a proportional integral control or a cascade control.

Key Words : Supply chain, Bullwhip effect, $z$-Transform, Batch ordering, Cascade control, Decimator, Expander

INTRODUCTION

During the last decade, supply chain management attracts a lot of attentions among the process system engineering researchers. One of many challenges in supply chain management is to control the inventory level and to improve custom satisfaction simultaneously. Meanwhile, one has to consider the control effect of the whole supply chain. “Bullwhip effect”, i.e., the magnification of amplitudes of demand perturbations from the tail to upstream levels of the supply chain also has to be reduced.

The ordering policy can be viewed as control strategy of its inventory level. Recently, Perea-López et al. (2000, 2001) examined the dynamic behavior of a supply chain system and analyzed the impact of several heuristic control laws, again using time-domain simulation. Most of the mathematical models on feedback control of supply chain systems are time-domain simulation models. For example, Porter and coworkers (Porter and Bradshaw, 1974; Bradshaw and Porter, 1975; Mak et al., 1976) analyzed the feedback control of a supply chain using time-domain simulation and D-partition analysis. However, for a discrete dynamic system, it is more convenient to apply linear control analysis in the $z$-transform domain. On the other hand, in many cases, the orders from any unit of a supply chain are based on batch policies, i.e., they correct the orders from their down stream for many days, and put its request to upper stream units only once. Previous researches in this area assume each unit of any supply chain put their orders to its upper stream any time, i.e., continuous ordering policy is assumed, but this is not true in most cases. Until now, there exists no researcher to derive a whole batch ordering model to analyze the dynamic behaviors of a supply chain.

In the literatures, there are many papers in studying the discrete consumer demand and batch ordering inventory policies. For example, Forsberg (1996, 1997) solved the two-level inventory systems with the method of Erland distribution. Recently,
Axsäter (1997, 1998, 2000, 2001) presented the exact analyses of two echelon inventory problems. In addition, Sachan (1984), Teng (1994, 1996) extended the classical economic order quantity to establish replenishment policies in order to find a minimum cost. Andersson and Marklund (2000) applied an approximate cost function to optimize the performances of the decentralized distribution system. Similarly, Downs et al. (2001) developed an order up to level inventory model using linear programming technique to handle the logistic problem. Cachon (1999, 2001) also used the order up to level policy to exactly evaluate the performances of a batch ordering inventory system. The above works all focused to minimize the systems cost by applying various optimization approach under the order up to level inventory policy. However the researches did not to propose an entire mathematical model to investigate the bullwhip effect for a batching order system, and this may damage the entire supply chain since the whole system may not work in stable.

The causes of the bullwhip effect were presented by Lee et al. (1997a, 1997b). They are demands forecasting, order batching, price fluctuation, lead time, and shortage gaming. Because the bullwhip effect distorts the information and results in the misguided decisions-making, it is valuable to develop a control strategy to reduce the bullwhip effect. Chen et al. (2000a, 2000b) quantified the bullwhip effect that is due to the effects of demand forecasting and lead time. Towill and coworkers (Dejonckheere et al., 2002; Towill, 1982) examined the role of demand forecasting in such systems using transfer function analysis. However, they have not discussed the subject in the context of feedback control. Chen et al. (2000b) discussed the merit of using exponential filter in forecasting, also in a feed-forward context.

In this study, we first present a batch ordering dynamic model using z-transform by multi-rate sampling approach to examine the actions of the supply chain system. A batch ordering supply chain model that is different from the continuous ordering is analyzed using z-transform. Analytical forms of the closed loop transfer functions are obtained. The causes of bullwhip become quite apparent using the model and stability analysis. A PI and a cascade control structures are proposed and controllers are synthesized to reduce the bullwhip effect.

DERIVATION OF DYNAMIC MODEL FOR SINGLE UNIT

Consider a simple supply chain as shown in Fig. 1. We assume the demand ways of the customer are continuous, then the target node lumps the all demands after a constant period $M$ to place a batch order to its upstream node. Now, let $I(t)$ denote the actual inventory of the target logistic node at any time instant $t$. For the target node, the amount of goods received from its upstream node is denoted by $Y_t$, and the number of products delivered to its downstream node is denoted by $Y_D(t)$. A time delay of $L$ is assumed for all delivery actions so that goods dispatched at time $t$ will arrive at time $t + L$. However, due to need for examination and administrative processing, this new delivery is only available to customer at $t + L + 1$. The inventory balance at target node is given by:

$$I(t) = I(t-1) + Y_t - Y_D(t).$$

Due to the delay in delivery, an inventory position $IP(t)$ is defined to better monitor the change in inventory:

$$IP(t) = IP(t-1) + Y_t - Y_D(t).$$

We assume that ordering information is communicated instantaneously. However, $D(t)$, which is defined as the total demands from the all downstream nodes at time $t$ will only be processed at time $t + 1$, again due to administrative delay. Therefore, a standing order for target node at time $t$, $O(t)$ is defined as the amount of order to be processed at time $t + 1$. Moreover, we assume that an order can be accumulated to the next time step if it is not fulfilled, since each customer has only one supplier in our simple supply chain. Therefore, the standing order for the target node at time $t$ is the sum of the order placed at time $t$, plus any the unfulfilled order:

$$O(t) = D(t) + O(t-1) - Y_D(t).$$

The actual delivery, corresponding to a control valve’s action, has physical limits. If there are enough inventory to satisfy the standing order at $t - 1$, all the orders will be delivered. Otherwise, the inventory will be cleared (i.e. the valve is fully open). Similarly, if the downstream node already has too much inventory, the supplier will just stop delivery (i.e. the valve is fully closed), return of goods is not taken into consideration. Therefore,

$$Y_D(t) = \begin{cases} 0 & O(t-1) \leq 0 \\ O(t-1) & 0 \leq O(t-1) \leq I(t-1) \\ I(t-1) & 0 \leq I(t-1) \leq O(t-1) \end{cases}$$

![Fig. 1.](image-url) A batch ordering supply chain with continuous demand.
For simplicity, let the customer satisfaction be represented by a backorder defined as the difference between the total standing order at \( t-1 \) and the amount of goods actually delivered at \( t \):

\[
BO(t) = O(t-1) - Y_D(t).
\]

The larger \( BO \), the poorer is the customer satisfaction.

The \( z \)-transform of the above discrete time model is given by

\[
I(z) = \frac{z}{z-1} (z^{-1} Y_U(z) - Y_D(z)),
\]

\[
IP(z) = \frac{z}{z-1} (Y_U(z) - Y_D(z)),
\]

\[
O(z) = \frac{z}{z-1} (D(z) - Y_D(z)),
\]

\[
Y_D(z) = \begin{cases} 0 & z^{-1} O(z) \leq 0 \cr z^{-1} O(z) & 0 \leq z^{-1} O(z) \leq z^{-1} I(z), \cr z^{-1} I(z) & 0 \leq z^{-1} I(z) \leq z^{-1} O(z) \end{cases},
\]

\[
BO(z) = \frac{-z}{z-1} Y_D(z).
\]

THE ANALYSIS OF A UNIT UNDER PROPORTIONAL ORDERING CONTROL POLICY

The objective of this section is to examine some cases of the supply chain operations with a proportional control of inventory levels. Of course, a real operation of each unit should be somewhere in between of these cases. However, such asymptotic analysis provides extremely useful insights. The downstream demands take place every unit time, but the target node places a batch order every \( M \) unit times.

A simple P-control can be used:

\[
U(t) = \begin{cases} K \times (SP(t) - IP(t)), & t = nM, n = 0, 1, 2, 3, \ldots, \\
0, & \text{otherwise}
\end{cases},
\]

where the superscripts \( M \) and \( E \) in Eq. (12) denote the decimator and expander respectively, so the term \( X^{ME} (X = SP, IP, \cdots) \) represents the sequence \( X(t), <t = 0, 1, 2, 3, \cdots> \) through \( M \)-fold decimator then proceeding \( E \)-fold expander.

Definitions of decimator and expander

An \( M \)-fold decimator (see Fig. 2(a)) for a sequence \( X(t) <t = 0, 1, 2, 3, \cdots> \) is defined as

\[
R^M(t) = X(M \times t),
\]

where \( M \) is a positive integer. So we can derive the \( M \)-fold decimator by taking a \( z \)-transform

\[
R^M(z) = \frac{1}{M} \sum_{k=0}^{M-1} X \left( z^{1/M} \exp \left( \frac{-2\pi ki}{M} \right) \right), \quad i = \sqrt{-1}.
\]

For example, if \( M = 2 \), then the inputs and outputs of the decimator are given by

\[
X(t) \quad X(0) \quad X(1) \quad X(2) \quad X(3) \quad X(4) \quad X(5) \quad X(6) \cdots
\]

\[
R^M(t) \quad X(0) \quad X(2) \quad X(4) \quad X(6) \cdots
\]

Similarly, an \( E \)-fold Expander (see Fig. 2(b)) for a sequence \( X(t) <t = 0, 1, 2, 3, \cdots> \) is defined as

\[
R^E(t) = \begin{cases} X(t/E), & \text{if } t \text{ is a multiple of } E \\
0, & \text{otherwise}
\end{cases},
\]

where \( E \) is a positive integer. So the \( z \)-transform of the \( E \)-fold decimator can be yielded as

\[
R^E(z) = X(z^E).
\]

If \( E = 2 \), then the inputs and outputs of the expander are given by

\[
X(t) \quad X(0) \quad X(1) \quad X(2) \quad X(3) \cdots
\]

\[
R^E(t) \quad X(0) \quad X(1) \quad X(2) \quad X(3) \cdots
\]

Fig. 2. (a) An \( M \)-fold decimator; (b) an \( E \)-fold expander.
Then, the interchanger of decimator and expander can be derived from following the above definitions. Firstly, if the sequence \( X(t) \) proceeds the \( M \)-fold decimator, then \( E \)-fold expander as depicted in Fig. 3(a), we denote \( R^{ME}(z) \) is the \( z \)-transform of the sequence
\[
R^{ME}(z) = \frac{1}{M} \sum_{k=0}^{M-1} X \left( z^{-E/M} \exp \left( \frac{-2\pi ki}{M} \right) \right). \tag{17}
\]

Secondly, it is contrary to the proceeding order in Fig. 3(a), we consider the configuration in which the \( E \)-fold expander proceeds the \( M \)-fold decimator as depicted in Fig. 3(b). The \( z \)-transform of the output signals is
\[
R^{EM}(z) = \frac{1}{M} \sum_{k=0}^{M-1} X \left( z^{-E/M} \exp \left( \frac{-2\pi ki}{M} \right) \right). \tag{18}
\]

**Stability**

In this section, we will examine the stability condition for several extreme cases, and the real cases for a node should be somewhere in between:

**Case 1: Infinitive high inventory position**

We assume that the upstream supplier has sufficient inventory so that the customer demands are always satisfied: i.e., \( Y_t(z) = z^{-1} U(z) \). Furthermore we assume that the set-point of the target node is sufficiently high so that there will always be sufficient inventory to satisfy all customer demands, i.e., \( Y_D(z) = z^{-1} O(z) = z^{-1} D(z) \). According to the above assumptions and combining the Eq. (12), then the inventory position at decimation time becomes
\[
\text{IP}^M(z) = \frac{K \times \text{SP}^M(z)}{z - 1 + K} - \frac{1}{z - 1 + K} \left( \frac{D(z)}{z - 1} \right)^M, \tag{19}
\]

with the characteristic equation
\[
H(z) = z - 1 + K = 0. \tag{20}
\]

\[\text{(a) } X(t) \rightarrow \downarrow M \rightarrow \uparrow E \rightarrow R^{ME}(t)\]
\[\text{(b) } X(t) \rightarrow \uparrow E \rightarrow \downarrow M \rightarrow R^{EM}(t)\]

Fig. 3. (a) An \( M \)-fold decimator proceeding \( E \)-fold expander; (b) an \( E \)-fold expander proceeding \( M \)-fold decimator.

A discrete system is stable if all the roots of the characteristic equation lie within the unit circle. According to this rule, we can get \( 0 \leq K \leq 2 \), thus the ultimate gain \( K_U = 2 \). In this condition, the gain must be smaller than 2, otherwise the system must be unstable.

**Case 2: Infinitive low inventory position**

If upstream supplier has also sufficient inventory, but the inventory position set-point of the target node is low so that there will always be insufficient inventory to satisfy all customer demands, i.e., \( Y_0(z) = z^{-1} R(z) \). Thus, the transfer function of inventory position at decimation time can be derived as
\[
\text{IP}^M(z) = \frac{K \times \text{SP}^M(z)}{z - 1 + K} - \frac{1}{z - 1 + K} \left( \frac{D(z)}{z - 1} \right)^M, \tag{21}
\]

with the characteristic equation
\[
H(z) = z^n + \frac{K(z^n - 1)}{z - 1} = 0. \tag{22}
\]

Therefore, if upstream supply is infinite and the inventory position set-point is so low that there is always insufficient inventory than standing order. Moreover, the closed loop transfer function (Eq. (21)) is independent of customer demands \( D(z) \). Therefore, when there is unlimited supply upstream supply but a low stock target, the inventory position becomes independent of fluctuations in downstream demands. In addition, basing on the criterion of the stability that the stability range is \( 0 \leq K \leq 1 \), so the ultimate gain \( K_U = 1 \).

**Bullwhip effect**

The bullwhip effect can be represented as amplification of demand fluctuations from downstream to upstream. When there is sufficient supply and high stock, substituting the Eq. (12) into Eq. (19), we get
\[
\text{UM}(z) = \frac{K \times (z - 1)}{z - 1 + K} - \frac{1}{z - 1 + K} D^M(z), \tag{23}
\]

where the term
\[
D^M(z) = (z - 1) \left( \frac{D(z)}{z - 1} \right)^M. \tag{24}
\]

This equation represents the summation of all customer demands during the period of \( M \). That is to say, the notation \( D^M(z) \) denotes the \( z \)-transform of \( D(t) \), it can be defined as
\[ \bar{D}(t_M) = \sum_{k=0}^{M-1} D((t_M - 1) \times M + k). \]  

So, \( t_M \) is a positive integer number, we call it a “decimation time”.

One factor that “bullwhip” is usually attributed to is aggressive ordering. We have demonstrated the system would become unstable when \( K_U = 2 \), the ultimate gain. Here, we show that when there is no change in inventory position set point, “bullwhip” effect is found only if the controller gain \( K_J \) is set greater than 1.

Assuming that there is no change in set point, the ratio of orders to successive nodes can be expressed as:

\[ \frac{|U^M(z)|}{|D^M(z)|} = \frac{K}{|z-1+K|}. \]  

The amplitude in demand fluctuations will be amplified if:

\[ \frac{|U^M(z)|}{|D^M(z)|} = \frac{K}{|e^{i\omega} - 1 + K|} > 1 \quad \forall \omega. \]  

By mathematic manipulations, the condition is met only if \( K > 1 \). It shows that bullwhip is mainly caused by high frequency fluctuations in customer demands when \( K > 1 \), i.e., the manager of the distributing node responded too aggressively to short-term fluctuations. If \( K < 1 \), the magnitude ratio can actually be reduced along the chain.

### CLASSICAL CONTROL WITH DEMAND FORECASTING

If we attempt to forecast the customer demands and set the inventory position target accordingly, as shown in Fig. 4, the closed loop responses of inventory position \( IP \) and order to supplier \( U \) at the decimation point become:

\[ IP^M(z) = \frac{\beta \times F(z) \times C(z) - 1}{z-1 + C(z)} D^M(z), \]  

\[ U^M(z) = \frac{C(z) \times (\beta \times F(z) \times (z-1) + 1)}{z-1 + C(z)} D^M(z), \]  

where \( C(z) \) is the controller, \( F(z) \) is the forecaster used to predict the current demand, and \( \beta \) is the function of lead time \( L \) and decimation number \( M \). Chen et al. (2000a) suggested the use of exponential filter:

\[ F(z) = \frac{\alpha}{z + \alpha - 1}. \]  

### P-only controller

If the controller \( C(z) = K \) in Eqs. (28) and (29) as depicted in Fig. 4, it becomes a P-only controller mode. From the stability analysis in the previous section, we demonstrate the proportional gain must be smaller than two. Obeying this limitation we use various gains (\( K = 0.80, 0.85, 0.90 \)) to estimate the variance ration of order (\( U \)) to demand (\( D \)). The variance ratio, i.e., \( \text{Var}(U)/\text{Var}(D) \), if its value is bigger than one that means the bullwhip effect appears in this unit. As shown in Fig. 5, when simulating this result, we assume that the customer demand is stochastic, i.e., \( d \in N(m, \sigma) \). Here we use \( m = 20, \sigma = 4 \), and the other parameters related to the simulations in this study are given as follows, \( \alpha = 0.1, \beta = 1.5, M = 5 \), and \( L = 1 \). From Fig. 5 the variance ratio is bigger than 1.0 while the gain \( K \) is not smaller than 0.9. That is to say, if we use P-control mode with demand forecasting, the bullwhip effect can be reduced while \( K < 0.9 \). However, Figs. 6 and 7 show that there is a big offset between the set point and inventory position. This offset will result in the accumulation of a large of backorder and dissatisfactory customer service. The offset is inevitable for a P-only controller. In order to avert this flaw in customer satisfaction, a PI controller should be implemented.

### PI controller

A PI controller algorithm in the type of \( z \)-transformation is written as

\[ C(z) = K \left( 1 + \frac{1}{\tau z - 1} \right), \]  

where \( \tau \) is the integer constant.
Figure 8 presents the simulation result of the ratio of the variance of a unit under demands forecasting. In case of $\frac{\text{Var}(U)}{\text{Var}(D)} > 1$, the bullwhip effect appears in this unit. Figure 8 proposes that $K = 0.7$, $\tau = 5$ should be implemented to avoid the bullwhip effect for all decimation time. As shown in Fig. 9, the offset has been eliminated and the bullwhip effect has also diminished by using a PI controller with $K = 0.7$ and $\tau = 5$, and compared Fig. 10 with Fig. 7, the backorder (customer satisfaction) is completely eliminated by the PI controller. Note that the fluctuations in the inventory position at decimation time still exist. In other words, the long term trends in customer demands can be more progressive by the further order control policies. Nevertheless, a better trend in customer demands and a less bullwhip effect for a supply chain system are oppositional goal to each other. We will implement a cascade control algorithm to coordinate this conflict in the following section.

Fig. 5. Estimation of the variance ratio of $\frac{U}{D}$ with some different $K$ values of P-only controller.

Fig. 6. Simulation results of P-controller ($K = 0.8$) with demands forecasting at decimation time.

Fig. 7. Dynamic simulation results of inventory position and backorder for a P-controller ($K = 0.8$) with demands forecasting at real time.

Fig. 8. Estimation of the variance ratio of $\frac{U}{D}$ with some different $K$ values of PI controller and the same value of $\tau = 5$.

Fig. 9. Simulation results of PI controller ($K = 0.7$, $\tau = 5$) with demands forecasting at decimation time.
Fig. 10. Dynamic simulation results of inventory position and backorder for a PI-controller ($K = 0.7$, $\tau = 5$) with demands forecasting at real time.

**CASCADE CONTROL MODE**

A cascade control scheme is shown in Fig. 11 that is composed of a P-only controller and a PI controller. The closed loop transfer functions are given by

$$IP^M(z) = \frac{\beta F(z) \times CC(z) \times CC(z) - 1}{1 + F(z) \times CC(z)} D^M(z),$$

(32)

$$U^M(z) = \frac{C(z) \times (\beta \times (z - 1) \times F(z) \times CC(z) + F(z) \times CC(z) + 1)} {1 + C(z) \times (1 + CC(z) \times F(z))} D^M(z).$$

(33)

If an exponential filter with $\alpha = 0.1$ is used for the forecaster $F(z)$, too. And two medium gains of $K = 0.7$ and 0.8 are used for the inner loop. The following PI cascade controller is used

$$CC(z) = K_C \times \left(1 + \frac{1}{\tau_C \times z - 1}\right).$$

(34)

With the same value of $\tau_C = 5$, and the outer loop gains of $K_C = 0.8$ and 1.2 are used to calculate the variance ratios as plotted in Fig. 12. This graph reveals the smaller variability in the place-order action. Namely, the bullwhip effect can be lessened more effectively by utilizing a cascade control than a PI control mode. Likewise, the responses of the inventory position display that the tracks of the inventory set point (Fig. 13) can be pursue very well. The simulation result (Fig. 14) that there is no backorder demonstrates the high customer satisfaction. In summary, the bullwhip effect reduction and customer satisfaction improvement are both improved by using the cascade control ordering strategy.
Fig. 14. Dynamic simulation results of inventory position and backorder for a cascade controller ($K = 0.7$, and $K_C = 0.8$, $\tau_C = 5$) with demand forecasting at real time.

CONCLUSION

In this study we propose a dynamic discrete model for a batch replenishment ordering supply chain system. In aiming at the supply chain system of a batch ordering with continuous customer demands, it is the first paper to derive a mathematical $z$-transformation model by signal processing techniques. Thus, the characteristic equations of the closed loop transfer functions are obtained. This approach helps us to analyze easily the stability of the supply chain system. Further, the bullwhip effect can be investigated. The study proves that bullwhip effect is inevitable if the standard heuristic ordering policy is employed with demand forecasting. Several alternative ordering policies were formulated as P-only, PI and cascade control schemes. By implementing a PI controller, the good performances of the dynamic behaviors can be reached in terms of the bullwhip effect reduction and customer satisfaction improvement. The offset between the set point and inventory level is also eliminated. We further derive a cascade control scheme that not only provides efficient control of the inventory position of a supply chain unit without causing bullwhip effect, but raises the customer satisfaction by providing more active tracking of the customer demand.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$BO$</td>
<td>backorder, number of item</td>
</tr>
<tr>
<td>$C$</td>
<td>the transfer function of controller</td>
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<tr>
<td>$D$</td>
<td>customer demand, number of item</td>
</tr>
<tr>
<td>$\bar{D}$</td>
<td>summation of the customer demands</td>
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<td>$E$</td>
<td>the total customer demands at decimation time domain, number of item</td>
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<tr>
<td>$E^M$</td>
<td>$E$-fold expander</td>
</tr>
<tr>
<td>$F$</td>
<td>exponential filter</td>
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<tr>
<td>$I$</td>
<td>actual inventory, number of item</td>
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<tr>
<td>$IP$</td>
<td>inventory position, number of item</td>
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<td>$K$</td>
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<td>proportional gain of the other loop</td>
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<td>$K_U$</td>
<td>ultimate gain</td>
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<td>mean</td>
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<td>time, day</td>
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<tr>
<td>$U$</td>
<td>the amount of order, number of item</td>
</tr>
<tr>
<td>$U^M$</td>
<td>the amount of order at decimation time domain, number of item</td>
</tr>
<tr>
<td>$Y_D$</td>
<td>the number of products delivered to the customer from the target node, number of item</td>
</tr>
<tr>
<td>$Y_U$</td>
<td>the number of products from the supplier delivered to the target node, number of item</td>
</tr>
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Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$\alpha$</td>
<td>parameter of filter</td>
</tr>
<tr>
<td>$\beta$</td>
<td>a variable that is function of $M$ and $L$</td>
</tr>
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<td>$\sigma$</td>
<td>variance</td>
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<td>$\tau$</td>
<td>integral constant of a PI</td>
</tr>
<tr>
<td>$\tau_C$</td>
<td>integral constant of a cascade control</td>
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REFERENCES


Axsäter, S., “Exact Analysis of Continuous Review (R,Q) Policies in Two-Echelon Inventory Systems with Com-

supply chain management is one of the most popular topics in recent years. Many scholars and experts have studied this area. The focus of this field of study is on the inventory replenishment strategy, which is to reduce inventory costs and reduce the so-called Bullwhip effect. One way to achieve this is to use system theory to determine the replenishment strategy. This paper will use this theoretical concept to develop a discrete dynamic model for the replenishment strategy. Then, the mathematical method of signal processing is used to convert the model into a z-transform type to clearly describe the operation behavior of the supply chain and explain the factors that cause system instability and the main cause of the Bullwhip effect. It not only proves the dynamic behavior of the system mathematically but also simulates the results by programming.

The main cause of the Bullwhip effect is the demand variation, and the replenishment strategy is another important factor.